CU-TP/01-05

Mixing among light scalar mesons and $L = 1$ $q\bar{q}$ scalar mesons

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Abstract

Following the re-establishment of the $\sigma(600)$ and the $\kappa(900)$, the light scalar mesons $a_0(980)$ and $f_0(980)$ together with the $\sigma(600)$ and the $\kappa(900)$ are considered as the chiral scalar partner of pseudoscalar nonet in $SU(3)$ chiral symmetry, and the high mass scalar mesons $a_0(1450)$, $K_0^*(1430)$, $f_0(1370)$ and $f_0(1710)$ turned out to be considered as the $L = 1$ $q\bar{q}$ scalar mesons. We assume that the high mass of the $L = 1$ $q\bar{q}$ scalar mesons is caused by the mixing with the light scalar mesons. For the structure of the light scalar mesons, we adopted the $qq\bar{q}\bar{q}$ model in order to explain the "scalar meson puzzle". The inter-mixing between the light scalar nonet and the high mass $L = 1$ $q\bar{q}$ nonet and the intra-mixing among each nonet are analyzed by including the glueball into the high mass scalar nonet.

PACS number(s): 11.30.Rd, 12.39.Mk, 14.40.-n

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1 INTRODUCTION

The σ meson was introduced theoretically in connection with the linear sigma model (LσM)[1] and listed in the Particle Data Group (PDG) until 1972. This meson disappeared in PDG until 1996, because this resonance was not narrow and the broad resonance could not easily be distinguished from a background. In recent analyses [2] of $\pi\pi$ and πK scattering phase shift, the scalar mesons, $\sigma(600)$ with $I = 0$ and $\kappa(900)$ with $I = 1/2$, have been observed. These analyses use the interfering Breit-Wigner amplitude and introduce a negative background phase of a hard core type.

In connection with the re-establishment of these particles, they and already established iso-singlet $f_0(980)$ and iso-triplet $a_0(980)$ lower than 1 GeV turned out to be considered as a chiral partner (σ-nonet) of the pseudoscalar (π-nonet) in $SU(3)$ chiral symmetry [3]. Many authors analyzed the $f_0(980)$ and $a_0(980)$ mesons using the $K\bar{K}$ molecule model [4] or $qq\bar{q}\bar{q}$ model [5] rather than the above LσM in order to explain the so-called "scalar meson puzzle" why the $f_0(980)$ degenerate to the $a_0(980)$ has so large $K\bar{K}$ decay width compared to the $\pi\pi$ decay width. Anyway, the light scalar mesons with masses lower than 1 GeV are considered as not the conventional $L = 1$ $q\bar{q}$ scalar nonet by many authors. The high mass scalar mesons $a_0(1450)$, $K_0^*(1430)$, $f_0(1370)$ and $f_0(1710)$ turn out to be classified to the conventional $L = 1$ $q\bar{q}$ $SU(3)$ nonet.

However, here we encounter a puzzle: why $L = 1$ $q\bar{q}$ scalar mesons considered above have so high masses compared with other $L = 1$ $q\bar{q}$ 1^{++} and 2^{++} mesons. If $L \cdot S$ interaction is assumed, there must be satisfied a relation for masses of them: $m^2(2^{++}) - m^2(1^{++}) = 2(m^2(1^{++}) - m^2(0^{++}))$. But, experimentally, this relation is not satisfied at all: $m^2(1^{++})$ is less than $m^2(0^{++})$ as $m_{a_2} = 1318$MeV, $m_{a_1} = 1230$MeV, $m_{a_0} = 1474$MeV, and $m_{K_2^*} = 1429$MeV, $m_{K_1} = 1339$MeV, $m_{K_0^*} = 1412$MeV, where we used the average value of $K_1(1^{-+})$ and $K_1(1^{++})$ for the m_{K_1}. In order to solve this puzzle, we assume an inter-mixing between the light scalar nonet and the $L = 1$ $q\bar{q}$ nonet [6]. Because the difference between the mass of $m_{a_0} = 1474$MeV and the mass 1236MeV predicted

from the relation $m^2(2^{++}) - m^2(1^{++}) = 2(m^2(1^{++}) - m^2(0^{++}))$ is very large, the inter-mixing between the light scalar meson and the $L = 1$ $q\bar{q}$ meson is considered to be very strong. The strength of the mixing depends on the structure of scalar mesons. As seen later, although we assume the structure of the light scalar mesons to be $qq\bar{q}\bar{q}$, the mixing strength can be very large.

There are another important mixing between two isoscalar mesons in both light scalar nonet and $L = 1$ $q\bar{q}$ nonet. We call the mixing "intra-mixing". Only the intra-mixing is considered in many analyses of masses and decays of the light scalar nonet and the $L = 1$ $q\bar{q}$ nonet without considerations of the inter-mixing. However, as seen later, the strength of the inter-mixing is considered to be rather large compared with the strength of the intra-mixing. Furthermore, the existence of the scalar glueball is suggested in QCD [7] and the $f_0(1500)$ is considered to be the most probable candidate for scalar glueball [8]. Then the mixing between the $I = 0$ $L = 1$ $q\bar{q}$ meson and the scalar glueball has to be considered, because the mixing between the $L = 1$ $q\bar{q}$ meson and the scalar glueball is considered to be rather large compared with the mixing among the $I = 0$ mesons. Thus we will study the overall mixing; inter-mixing, intra-mixing and glueball mixing, in order to analyze the masses and decays of the light scalar mesons and $L = 1$ $q\bar{q}$ mesons.

In section 2, we will discuss about the fact that the strength of the inter-mixing is very large. In section 3, we discuss the structure of the light scalar mesons. In section 4, we will analyze the overall mixing; inter-mixing, intra-mixing and glueball mixing.

2 Strength of inter-mixing

In this section, we estimate the strength of the inter-mixing between the light scalar mesons and the conventional $L = 1$ $q\bar{q}$ scalar mesons. For the estimation of the inter-mixing, it is necessary to know the masses before mixing of the scalar mesons. The mass

values of the 2^{++} and 1^{++} mesons cited in Particl Data Group [9] are as follows;

$$\left\{\begin{array}{l} m_{a_2(1320)}=1318\mathrm{MeV},\quad m_{K_2^*(1430)}=1429\mathrm{MeV},\\ \qquad\qquad m_{f_2(1270)}=1275\mathrm{MeV},\quad m_{f_2'(1525)}=1525\mathrm{MeV},\\ m_{a_1(1260)}=1230\mathrm{MeV},\quad m_{K_1(1270/1400)}=1339\mathrm{MeV},\\ \qquad\qquad m_{f_1(1285)}=1282\mathrm{MeV},\quad m_{f_1(1420)}=1426\mathrm{MeV},\end{array}\right. \tag{1}$$

where we averaged the $m_{K_1(1270)}=1273$MeV and $m_{K_1(1400)}=1402$MeV in quadratic mass because they are considered to be mixed up. We plot the $L=1$ $q\bar{q}$ 2^{++}, 1^{++} and 0^{++} mass spectra in Fig. 1. Because the mass differences between $m_{a_2(1320)}$, $m_{f_2(1270)}$ and beween $m_{a_1(1260)}$, $m_{f_1(1285)}$ are very small, we consider the ideal mixing limit, neglect these mass differences and average these masses as

$$\left\{\begin{array}{l} m_{a_2(1320)}=m_{f_2(1270)}=1297\mathrm{MeV},\ m_{K_2^*(1430)}=1429\mathrm{MeV},\ m_{f_2'(1525)}=1525\mathrm{MeV},\\ \\ m_{a_1(1260)}=m_{f_1(1285)}=1256\mathrm{MeV},\ m_{K_1}=1339\mathrm{MeV},\ m_{f_1(1420)}=1426\mathrm{MeV}.\end{array}\right. \tag{2}$$

If we assume the $L\cdot S$ force for $L=1$ $q\bar{q}$ bound states, then the following well-known mass relation is obtained,

$$m^2(2^{++})-m^2(1^{++})=2(m^2(1^{++})-m^2(0^{++})). \tag{3}$$

From this relation, the masses of $L=1$ $q\bar{q}$ 0^{++} mesons before inter-mixing denoted by $\overline{a_0(1450)}$, $\overline{f_0(1370)}$, $\overline{K_0^*(1430)}$ and $\overline{f_0(1710)}$ are estimated as follows;

$$\begin{array}{l} m_{\overline{a_0(1450)}}=m_{\overline{f_0(1370)}}=1236\mathrm{MeV},\\ m_{\overline{K_0^*(1430)}}=1307\mathrm{MeV},\quad m_{\overline{f_0(1710)}}=1374\mathrm{MeV}\end{array} \tag{4}$$

where the ideal (intra-)mixing mass relation $m^2_{\overline{f_0(1710)}}=2m^2_{\overline{K_0^*(1430)}}-m^2_{\overline{a_0(1450)}}$ is used.

Because the mass $m_{\overline{a_0(1450)}}$ for $I=1$ $a_0(1450)$ before inter-mixing is 1236MeV and the mass for light scalar $I=1$ meson $a_0(980)$ is 985MeV, the $m_{\overline{a_0(980)}}$ for $a_0(980)$ before inter-mixing is estimated as 1271MeV using the mass relation for 2-body mixing $m^2_{a_0(1450)}-m^2_{\overline{a_0(1450)}}=m^2_{\overline{a_0(980)}}-m^2_{a_0(980)}$. Similarly, the mass $m_{\overline{\kappa(900)}}$ for $I=1/2$ $\kappa(900)$ before inter-mixing to be 1047MeV is obtained from the masses $m_{\kappa(900)}=900$MeV, $m_{K_0^*(1430)}=1412$MeV and $m_{\overline{K_0^*(1430)}}=1307$MeV. The mass $m_{\overline{\sigma(600)}}$ for $\sigma(600)$ before inter-mixing is obtained as 760MeV from the ideal (intra-)mixing mass relation $m^2_{\overline{\sigma(600)}}=2m^2_{\overline{\kappa(900)}}-m^2_{\overline{a_0(980)}}$.

$$\begin{array}{l} m_{\overline{a_0(980)}}=m_{\overline{f_0(980)}}=1271\mathrm{MeV},\\ m_{\overline{\kappa_0(900)}}=1047\mathrm{MeV},\quad m_{\overline{\sigma(600)}}=760\mathrm{MeV}.\end{array} \tag{5}$$

We can easily estimate the strength of inter-mixing for the $I = 1$ $a_0(1450)$ and $a_0(980)$ and $I = 1/2$ $K_0^*(1430)$ and $\kappa(900)$ because these states have no effects from the intra-mixing. If we express the transition strength between $\overline{a_0(980)}$ and $\overline{a_0(1450)}$ as λ_{a_0}, then the mass matrix is written as

$$\begin{pmatrix} m^2_{\overline{a_0(980)}} & \lambda_{a_0} \\ \lambda_{a_0} & m^2_{\overline{a_0(1450)}} \end{pmatrix}, \quad m_{\overline{a_0(980)}} = 1.271\text{GeV}, \quad m_{\overline{a_0(1450)}} = 1.236\text{GeV} \tag{6}$$

and this has the eigenvalues $m_{a_0(980)} = 0.985\text{GeV}$ and $m_{a_0(1450)} = 1.474\text{GeV}$ at the $\lambda_{a_0} = 0.600\text{GeV}^2$. Mixing angle is evaluated as $\theta_{a_0} = 47.1°$. Similarly, for $I = 1/2$ scalar mesons, mixing matrix

$$\begin{pmatrix} m^2_{\overline{\kappa(900)}} & \lambda_{K_0} \\ \lambda_{K_0} & m^2_{\overline{K_0^*(1430)}} \end{pmatrix}, \quad m_{\overline{\kappa(900)}} = 1.047\text{GeV}, \quad m_{\overline{K_0^*(1430)}} = 1.307\text{GeV} \tag{7}$$

has eigenvalues $m_{\kappa(900)} = 0.900\text{GeV}$ and $m_{K_0^*(1430)} = 1.412\text{GeV}$ at the $\lambda_{K_0} = 0.507\text{GeV}^2$ and mixing angle $\theta_{K_0} = 29.5°$.

The obtained transition strength has two characteristics: (1) The strength is very large and (2) the values λ_{a_0} for $I = 1$ and λ_{K_0} for $I = 1/2$ are nearly equal. These characteristics are explained easily by assuming the structure of light scalar mesons which is explained explicitly in next section.

3 Structure of light scalar mesons

Because the scalar mesons are considered as an $SU(3)$ nonet, we present the scalar meson field as N'^b_a for conventional $L = 1$ $q\bar{q}$ scalar mesons and N^b_a for light scalar mesons. In representation N'^b_a, a and b denote the $SU(3)$ indices of triplet quark field q_a and unti-triplet quark field $\bar{q}^b$ respectively.

$$N'^b_a \sim q_a \bar{q}^b \quad \text{for } L = 1 \ q\bar{q} \text{ scalar mesons.} \tag{8}$$

For N^b_a, a and b are considered as $SU(3)$ indices of q_a and $\bar{q}^b$ when the light scalar mesons are considered as chiral scalar partner of pseudoscalar nonet with $q\bar{q}$ structure. In the case where the light scalar mesons are considered as $qq\bar{q}\bar{q}$, a and b in representation N^b_a denote

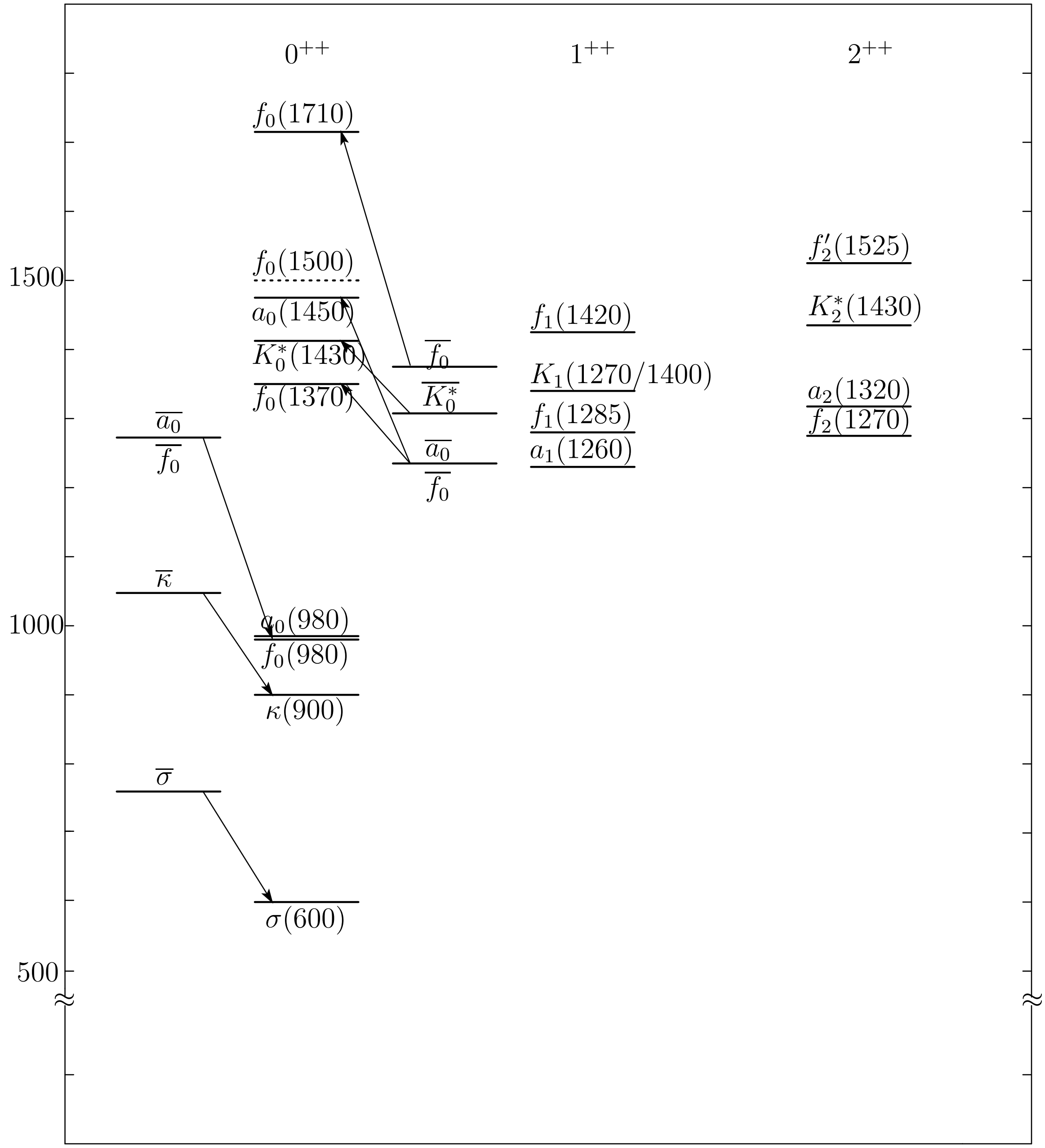

Figure 1: The 2^{++}, 1^{++} and 0^{++} meson mass spectra. The particles with overline are particles before-mixing with masses estimated using the masses of 0^{++}, 1^{++} and 2^{++} mesons. (See the text).

the $SU(3)$ indices of "dual" quark $T_a = \epsilon_{abc}\bar{q}^b\bar{q}^c$ and "dual" anti-quark $\bar{T}^a = \epsilon^{abc}q_bq_c$, respectively[5, 10].

$$N_b^a \sim q_b\bar{q}^a \quad \text{for } q\bar{q} \text{ light scalar mesons,} \tag{9}$$

$$N_b^a \sim T_b\bar{T}^a \sim \epsilon_{bde}\bar{q}^d\bar{q}^e\epsilon^{abc}q_bq_c \quad \text{for } qq\bar{q}\bar{q} \text{ light scalar mesons.} \tag{10}$$

Here, we show the explicit flavor configuration of the $qq\bar{q}\bar{q}$ scalar mesons to see what content of quarks are included in each scalar mesons,

$$\begin{array}{ccccc}
a_0^+ & \sim & N_1^2 & \sim & \bar{s}\bar{d}us \\
a_0^0 & \sim & \frac{1}{\sqrt{2}}(N_1^1 - N_2^2) & \sim & \frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds - \bar{s}\bar{u}us) \\
a_0^- & \sim & N_2^1 & \sim & \bar{s}\bar{u}ds \\
\kappa^+ & \sim & N_1^3 & \sim & \bar{s}\bar{d}ud \\
\kappa^0 & \sim & N_2^3 & \sim & \bar{s}\bar{u}ud \\
\overline{\kappa}^0 & \sim & N_3^2 & \sim & \bar{u}\bar{d}us \\
\kappa^- & \sim & N_3^1 & \sim & \bar{u}\bar{d}ds \\
f_0 & \sim & \frac{1}{\sqrt{2}}(N_1^1 + N_2^2) & \sim & \frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us) \\
\sigma & \sim & N_3^3 & \sim & \bar{u}\bar{d}ud
\end{array} \tag{11}$$

in the ideal mixing limit. Here, we assume that the state f_0 is $\frac{1}{\sqrt{2}}(N_1^1 + N_2^2) \sim \frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us)$ and σ is $N_3^3 \sim \bar{u}\bar{d}ud$ from the following consideration of masses of light scalar mesons.

We assume that the masses of light scalar mesons are described by the following chiral symmetric effective Lagrangian density

$$L^{eff} = -a\mathrm{Tr}(NN) - b\mathrm{Tr}(NNM) - \frac{1}{2}\lambda\mathrm{Tr}(N)\mathrm{Tr}(N), \tag{12}$$

where M is the "spurion matrix" $M = \mathrm{diag}[1, 1, x]$, x representing the symmetry breaking of the strange quark mass to non-strange quark mass. This formula is adopted in Black *et al* 's analysis [10]. This Lagrangian is equivalent to the generalized mass matrix used in our previous analysis of $q\bar{q}$ scalar mesons [11]. If light scalar mesons are $q\bar{q}$ mesons, the third term $\lambda\mathrm{Tr}(N)\mathrm{Tr}(N)$ denotes the transition amplitudes reproducing the $U(1)$ anomaly term in pseudoscalar meson case corresponding to the violation term of OZI rule in 2nd order shown in the Fig. 2.

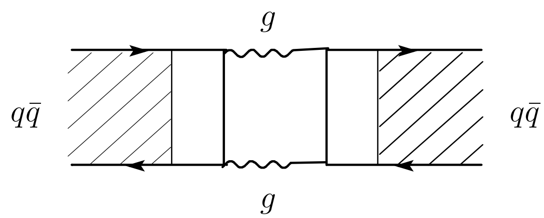

Figure 2: Graph for the $q\bar{q}$ meson transition violating the OZI rule in 2nd order. Straight lines represent the quarks and wavy lines the gluons. Oblique lines represent the gluon interactions in $q\bar{q}$ bound state.

We review the analysis for scalar meson masses using the effective Lagrangian (12) disregarding the inter-mixing. First, we assume that the $\sigma(600)$ is nearly equal to ideal state $\frac{N_1^1+N_2^2}{\sqrt{2}}$ and the $f_0(980)$ to ideal state N_3^3, because the $f_0(980)$ has the strong $s\bar{s}$ like structure. By using the Eq. (12), the scalar meson masses are represented as

$$m_a^2 = 2a + 2b,\ m_\kappa^2 = 2a + (1+x)b, \tag{13}$$

$$\begin{pmatrix} \sigma(600) \\ f_0(980) \end{pmatrix} = O \begin{pmatrix} \frac{N_1^1+N_2^2}{\sqrt{2}} \\ N_3^3 \end{pmatrix} \sim \begin{pmatrix} \frac{N_1^1+N_2^2}{\sqrt{2}} \\ N_3^3 \end{pmatrix},$$

$$O = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix},$$

$$O \begin{pmatrix} m_a^2 + 2\lambda & \sqrt{2}\lambda \\ \sqrt{2}\lambda & 2m_\kappa^2 - m_a^2 + \lambda \end{pmatrix} {}^tO = \begin{pmatrix} m_\sigma^2 & 0 \\ 0 & m_{f_0}^2 \end{pmatrix}. \tag{14}$$

From the relation $2m_\kappa^2 + 3\lambda = m_\sigma^2 + m_{f_0}^2$, we obtain $\lambda = -0.0999$ GeV^2 and eigenvalues 694MeV and 916MeV. One eigenvalues 694MeV of these is not so close to the values 600MeV, furthermore mixing angle is 64° and f_0 state is far from the ideal $s\bar{s}$ state. In Black *et al.*'s analysis , the $SU(3)$ breaking correction is introduced by adding the term $\mathrm{Tr}(NM)\mathrm{Tr}(NM)$ in Eq. (12) [10]. In our analysis [11], the $SU(3)$ breaking correction is introduced by multiplying k to λ as

$$O \begin{pmatrix} m_a^2 + 2\lambda & k\sqrt{2}\lambda \\ k\sqrt{2}\lambda & 2m_\kappa^2 - m_a^2 + k^2\lambda \end{pmatrix} {}^tO = \begin{pmatrix} m_\sigma^2 & 0 \\ 0 & m_{f_0}^2 \end{pmatrix}.$$

In this case, if $\lambda = -0.00935$ GeV^2 and $k = 5.48$, we can obtain the eigenvalues exactly equal to the values $m_\sigma = 600$MeV and $m_{f_0} = 980$MeV, but mixing angle is 83° and f_0 state is almost non-$s\bar{s}$ state. Thus, it is difficult to consider that the light scalar mesons are $q\bar{q}$ states.

Next we consider the case that the light scalar mesons are $qq\bar{q}\bar{q}$. In this case, the third term $\frac{1}{2}\lambda\mathrm{Tr}(N)\mathrm{Tr}(N)$ represents also the OZI violation terms in 2nd order as shown in Fig. 3. In this case, we adopt the quark configuration in Eq. (11) because the f_0 state

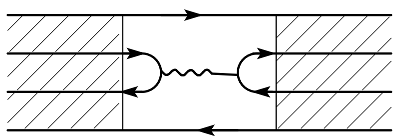

Figure 3: Graph for the $qq\bar{q}\bar{q}$ meson transition violating the OZI rule in 2nd order.

is almost $s\bar{s}$ like. The masses of this case are represented in the same form as the $q\bar{q}$ case except for the configuration of $f_0(980)$ and $\sigma(600)$ which are nearly $f_N \equiv \frac{N_1^1+N_2^2}{\sqrt{2}} \sim \frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us)$ and $f_S \equiv N_3^3 \sim \bar{u}\bar{d}ud$, respectively;

$$m_a^2 = 2a + 2b,\ m_\kappa^2 = 2a + (1+x)b, \tag{15}$$

$$\begin{pmatrix} f_0(980) \\ \sigma(600) \end{pmatrix} = O \begin{pmatrix} \frac{N_1^1+N_2^2}{\sqrt{2}} \\ N_3^3 \end{pmatrix} \sim \begin{pmatrix} \frac{N_1^1+N_2^2}{\sqrt{2}} \\ N_3^3 \end{pmatrix},$$

$$O = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix},$$

$$O \begin{pmatrix} m_a^2 + 2\lambda & \sqrt{2}\lambda \\ \sqrt{2}\lambda & 2m_\kappa^2 - m_a^2 + \lambda \end{pmatrix} {}^tO = \begin{pmatrix} m_{f_0}^2 & 0 \\ 0 & m_\sigma^2 \end{pmatrix}. \tag{16}$$

Here, it should be noted that the first and second terms in effective Lagrangian (12) contain the contributions corresponding to the transition represented by the graph in Fig (3). In fact, the masses for a_0^+, κ^+, f_N, f_S mesons and transition amplitude from f_N to f_S are represented as

$$\begin{aligned}
m_{a_0^+}^2 &=< \bar{s}\bar{d}us|H|\bar{s}\bar{d}us >= 2a + 2b = M_0 + 2(m_u + m_s) + < \bar{u}u|\bar{u}u > \lambda_{ss}, \\
m_{\kappa^+}^2 &=< \bar{s}\bar{d}ud|H|\bar{s}\bar{d}ud >= 2a + (1+x)b = M_0 + 3m_u + m_s + < \bar{s}u|\bar{s}u > \lambda_{uu}, \\
m_{f_N}^2 &=< \frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us)|H|\frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us) >= 2a + 2b + 2\lambda \\
&\qquad = M_0 + 2(m_u + m_s) + < \bar{u}u|\bar{u}u > \lambda_{ss} + 2 < \bar{s}s|\bar{s}s > \lambda_{uu}, \\
m_{f_S}^2 &=< \bar{u}\bar{d}ud|H|\bar{u}\bar{d}ud >= 2a + 2bx + \lambda = M_0 + 4m_u + 2 < \bar{u}u|\bar{u}u > \lambda_{uu}, \\
\sqrt{2}\lambda &=< \frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us)|H|\bar{u}\bar{d}ud >= \sqrt{2} < \bar{u}u|\bar{u}u > \lambda_{su},
\end{aligned} \tag{17}$$

where M_0, m_u and m_s are the terms proportional to the potential energy, $u(d)$ quark mass and s quark mass, respectively and last term represents the contribution of the OZI violation term in 2nd order. In obtaining Eq. (16), we assumed that the $SU(3)$ symmetry breaking is negligible for the contribution of the OZI violation term and then $<\bar{u}u|\bar{u}u>\lambda_{uu}=<\bar{s}u|\bar{s}u>\lambda_{uu}=<\bar{s}s|\bar{s}s>\lambda_{uu}=<\bar{u}u|\bar{u}u>\lambda_{ss}$ were assumed.

Diagonalising the mass matrix (16), we obtain the mixing angle as 26° and $\lambda = -0.0999$ GeV^2 and f_0 nearly equal to $f_N = \frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us)$ is obtained. To obtain the masses of $f_0(980)$ and $\sigma(600)$ exactly equal to 980 MeV and 600 MeV, we consider the inter-mixing in next section. From the analysis for the masses of light scalar mesons, we can conclude that the light scalar mesons prefer the $qq\bar{q}\bar{q}$ structure to $q\bar{q}$ one. Lastly, it should be noted that the strength of the transition amplitude $|\lambda| = 0.0999$ GeV^2 is rather small compared with that of the inter-mixing transition amplitude $|\lambda_{a_0}| = 0.60$ GeV^2. This is understood from the suppression effect of the OZI violation; the inter-mixing is OZI 1st order suppression and the intra-mixing is OZI 2nd order suppression.

4 Inter-, intra- and glueball mixing

We assume that the inter-mixing between the light scalar mesons N and $L=1$ $q\bar{q}$ mesons N' are represented as

$$\begin{aligned}
L_{01}^{eff} &= -\lambda_{01}\epsilon^{abc}\epsilon_{dec}N_a^d N_b'^e = \lambda_{01}(\mathrm{Tr}(NN') - \mathrm{Tr}(N)\mathrm{Tr}(N')) \\
&= \lambda_{01}[a_0^+ a_0'^- + a_0^- a_0'^+ + a_0^0 a_0'^0 + \kappa^+ K_0^{*-} + \kappa^- K_0^{*+} \\
&\quad + \kappa^0 \overline{K}_0^{*0} + \overline{\kappa}^0 K_0^{*0} - f_N f_N' - \sqrt{2} f_S f_N' - \sqrt{2} f_N f_S'], \qquad (18)
\end{aligned}$$

where it should be noted that the quark configurations of $I=0$ mesons are $f_N' = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$ and $f_S' = \bar{s}s$ for N' but $f_N = \frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us)$ and $f_S = \bar{u}\bar{d}ud$ for N. These inter-mixing transitions are represented by the OZI rule violating term in only 1st order as shown in Fig. (4). The reason why we assume the transition $\mathrm{Tr}(NN') - \mathrm{Tr}(N)\mathrm{Tr}(N')$ instead of $\mathrm{Tr}(NN')$ is that only the amplitude $\mathrm{Tr}(NN')$ contains the transtion $< f_S|f_S' >=$

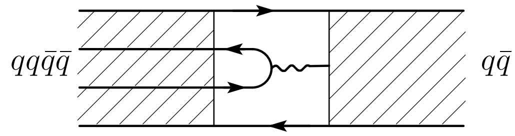

Figure 4: Graph for the inter-mixing transition between $qq\bar{q}\bar{q}$ and $q\bar{q}$. This graph violats the OZI rule in 1st order.

$< \bar{u}\bar{d}ud|\bar{s}s >$ which is the OZI rule violating term in 3rd order and then is negligible. The strentgthes of transition λ_{01} are estimated in section 2 as $\lambda_{a_0} = 0.60$ GeV^2, $\lambda_{K_0} = 0.51$ GeV^2 and the reason why the strength is so large and why λ_{a_0} and λ_{K_0} are nearly equal is undrestood easily from the Eq. (18).

In previous section, we considered the intra-mixing among the light scalar mesons using the mass matrix represented as

$$\begin{pmatrix} m_a^2 + 2\lambda_0 & \sqrt{2}\lambda_0 \\ \sqrt{2}\lambda_0 & 2m_\kappa^2 - m_a^2 + \lambda_0 \end{pmatrix}. \tag{16}$$

Because there is a glueball candidate $f_0(1500)$ [7] near the mass range of the $L = 1$ $q\bar{q}$ scalar mesons, it is necessary to consider the mixing between the glueball and the $L = 1$ $q\bar{q}$ scalar meson. We analyzed this problem in previous paper [11]. The masses of the $L = 1$ $q\bar{q}$ mesons before mixing and the transition amplitudes among the $L = 1$ $q\bar{q}$ mesons are described by the same effective Lagrangian as the light scalar mesons described in Eq. (12). The strength for the transition between the $L = 1$ $q\bar{q}$ meson and the glueball is described by the λ_G which corresponds to the graph shown in Fig. 5(a). In our previous analysis [11], we used the parameters λ_{GN} and λ_{GS} instead of the parameter λ_G considering the $SU(3)$ violation effect. The glueball mass before mixing is represented by the parameter λ_{GG} corresponding to the strength of the transition between pure glueball and pure glueball as shown in Fig. 5(b). In Ref. [11], we showed this as $m_G^2 + \lambda_{GG}$, where λ_{GG} in the expression represents the contribution from the diagram containing a quark loop in two gluon lines in Fig. 5(b). The mass matrix representing the intra-mixing among $L = 1$ $q\bar{q}$ mesons and glueball is

$$\begin{pmatrix} m_{a'}^2 + 2\lambda_1 & \sqrt{2}\lambda_1 & \sqrt{2}\lambda_G \\ \sqrt{2}\lambda_1 & 2m_{K'}^2 - m_{a'}^2 + \lambda_1 & \lambda_G \\ \sqrt{2}\lambda_G & \lambda_G & \lambda_{GG} \end{pmatrix}. \tag{19}$$

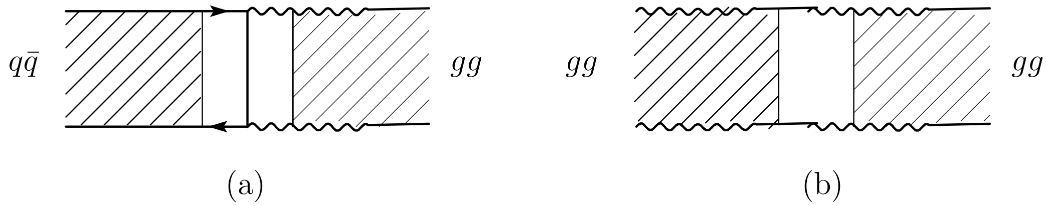

Figure 5: Graphs containing the glueball contributions. Fig. (a) represents the transition between $q\bar{q}$ meson and glueball gg. Fig. (b) represents the pure glueball-pure glueball transition.

We consider the overall mixing containing the inter-, intra- and glueball mixing represented by the mixing matrix O and mass matrix M as

$$\begin{pmatrix} f_0(980) \\ \sigma(600) \\ f_0(1370) \\ f_0(1710) \\ f_0(1500) \end{pmatrix} = O \begin{pmatrix} f_N \\ f_S \\ f'_N \\ f'_S \\ f_G \end{pmatrix}, \qquad O M\, {}^t O = M_D, \tag{20}$$

$$M = \begin{pmatrix} m_N^2 + 2\lambda_0 & \sqrt{2}\lambda_0 & \lambda_{01} & \sqrt{2}\lambda_{01} & 0 \\ \sqrt{2}\lambda_0 & m_S^2 + \lambda_0 & \sqrt{2}\lambda_{01} & 0 & 0 \\ \lambda_{01} & \sqrt{2}\lambda_{01} & m_{N'}^2 + 2\lambda_1 & \sqrt{2}\lambda_1 & \sqrt{2}\lambda_G \\ \sqrt{2}\lambda_{01} & 0 & \sqrt{2}\lambda_1 & m_{S'}^2 + \lambda_1 & \lambda_G \\ 0 & 0 & \sqrt{2}\lambda_G & \lambda_G & \lambda_{GG} \end{pmatrix},$$

$$M_D = \mathrm{diag}[m^2_{f_0(980)},\ m^2_{\sigma(600)},\ m^2_{f_0(1370)},\ m^2_{f_0(1710)},\ m^2_{f_0(1500)}].$$

We estimate the best fit values for λ_{01}, λ_0, λ_1, λ_G and λ_{GG} adopting the following mass values of m_N, m_S, $m_{N'}$ and $m_{S'}$ estimated in section 2;

$$\begin{aligned} m_N &= m_{\overline{a_0(980)}} = m_{\overline{f_0(980)}} = 1271\text{MeV}, \quad m_{\overline{\kappa(900)}} = 1047\text{MeV}, \\ m_S &= m_{\overline{\sigma(600)}} = 760\text{MeV}, \\ m_{N'} &= m_{\overline{a_0(1450)}} = m_{\overline{f_0(1370)}} = 1236\text{MeV}, \quad m_{\overline{K_0^*(1430)}} = 1307\text{MeV}, \\ m_{S'} &= m_{\overline{f_0(1710)}} = 374\text{MeV}, \end{aligned} \tag{21}$$

taking the least χ^2 defined by $\sum_a (m_a - m_{a_0})^2/\Delta m_a{}^2$, where m_a and Δm_a represent the experimental mass values and mass errors of the scalar meson a and m_{a_0} represents the value of the mass estimated. Estimated values of λ_{01}, λ_0, λ_1, λ_G and λ_{GG}, and $m_{f_0(980)}$,

$m_{\sigma(600)}$, $m_{f_0(1370)}$, $m_{f_0(1710)}$ and $m_{f_0(1500)}$ are as follows:

$$\begin{aligned}
&\lambda_{01} = 0.51\mathrm{GeV}^2, \quad \lambda_0 = 0.05\mathrm{GeV}^2, \quad \lambda_1 = 0.05\mathrm{GeV}^2, \\
&\lambda_G = 0.26\mathrm{GeV}^2, \quad \lambda_{GG} = 1.53\mathrm{GeV}^2, \\
&m_{f_0(980)} = 0.981(0.980/0.01)\mathrm{GeV}, \quad m_{\sigma(600)} = 0.455(0.600/0.10)\mathrm{GeV}, \\
&m_{f_0(1370)} = 1.376(1.350/0.05)\mathrm{GeV}, \quad m_{f_0(1710)} = 1.715(1.715/0.007)\mathrm{GeV}, \\
&m_{f_0(1500)} = 1.499(1.500/0.01)\mathrm{GeV},
\end{aligned} \tag{22}$$

where values in parentheses are (the experimental mass/the experimental error) of scalar mesons. Mixing matrix is obtained for estimated values of λ's,

$$\begin{pmatrix} f_0(980) \\ \sigma(600) \\ f_0(1370) \\ f_0(1710) \\ f_0(1500) \end{pmatrix} = \begin{pmatrix} 0.7129 & -0.3282 & -0.2223 & -0.5548 & 0.1640 \\ 0.1605 & 0.8402 & -0.5056 & -0.0604 & 0.0945 \\ 0.0625 & 0.4027 & 0.7000 & -0.5191 & -0.2729 \\ 0.5024 & 0.1550 & 0.4481 & 0.5221 & 0.5002 \\ -0.4580 & 0.0085 & 0.0639 & -0.3828 & 0.7997 \end{pmatrix} \begin{pmatrix} f_N \\ f_S \\ f_{N'} \\ f_{S'} \\ f_G \end{pmatrix}. \tag{23}$$

The fact that the estimated λ's have a character $\lambda_0 \sim \lambda_1 \ll \lambda_G \sim \lambda_{01}$ is consistent with the expected feature which comes from the OZI-suppressed character. We estimated the $\chi^2 = 2.40$ for this case. On the other hand, when we adopt the inter-mixing expressed by the effective Lagrangian $\mathrm{Tr}(NN')$ instead of $\mathrm{Tr}(NN') - \mathrm{Tr}(N)\mathrm{Tr}(N')$, we get the χ^2 values as $\chi^2 = 249.9$, which cannot be accepted. If we do not include the glueball into the overall mixing, the χ^2 value is $\chi^2 = 3.92$, then the glueball mixing with other scalar mesons is preferred to no glueball mixing in the scalar meson spectrum. From the estimated mixing parameters (23), we are able to predict the decay ratios and decay widths for the scalar meson decays to two pseudoscalar mesons ϕ's using the coupling among them as

$$\begin{aligned}
&\varepsilon^{abc}\varepsilon_{def} N_a^d \phi_b^e \phi_c^f \quad \text{for } N\phi\phi \text{ coupling, [10]} \\
&N_a'^b (\phi_b^c \phi_c^a + \phi_c^a \phi_b^c) \quad \text{for } N'\phi\phi \text{ coupling.}
\end{aligned} \tag{24}$$

Using these couplings, we will analyze the decay problem in the next work. Here, we want to restrict ourselves to comment that the mixing parameters estimated predict that the $f_0(980)$ has a large $s\bar{s}$ like character and the $\sigma(600)$ does very little $s\bar{s}$ like character as expected from the experiment.

5 Conclusion

Following the re-establishment of the $\sigma(600)$ and the $\kappa(900)$, the light scalar mesons $a_0(980)$ and $f_0(980)$ together with $\sigma(600)$ and $\kappa(900)$ are considered as the scalar nonet and on the other hand, the high mass scalar mesons $a_0(1450)$, $K_0^*(1430)$, $f_0(1370)$ and $f_0(1710)$ are turned out to be considered as the $L = 1$ $q\bar{q}$ scalar nonet. However, the masses of the $L = 1$ $q\bar{q}$ high mass scalar nonet are very large compared to other $L = 1$ $q\bar{q}$ 1^{++} and 2^{++} mesons. We assumed that the high mass of the $L = 1$ $q\bar{q}$ scalar nonet is caused by the mixing with the light scalar nonet.

In section 2, we estimated the masses of the $L = 1$ $q\bar{q}$ scalar nonet N' and light scalar nonet N before mixing as $m_{\overline{a_0(1450)}} = m_{\overline{f_0(1370)}} = 1236\text{MeV}$, $m_{\overline{K_0^*(1430)}} = 1307\text{MeV}$, $m_{\overline{f_0(1710)}} = 1374\text{MeV}$, $m_{\overline{a_0(980)}} = m_{\overline{f_0(980)}} = 1271\text{MeV}$, $m_{\overline{\kappa(900)}} = 1047\text{MeV}$, $m_{\overline{\sigma(600)}} = 760\text{MeV}$, using the well-known mass relation $m^2(2^{++}) - m^2(1^{++}) = 2(m^2(1^{++}) - m^2(0^{++}))$. The strength of inter-mixing are estimated from the $a_0(1450)$-$a_0(980)$ and $K_0^*(1430)$-$\kappa(900)$ mixings as $\lambda_{a_0} = 0.60\text{GeV}^2$ and $\lambda_{K_0} = 0.51\text{GeV}^2$. These are very large and nearly equal. These characters are recognized from the OZI-rule and the effective Lagrangian of the transition between N' and N discussed in section 4.

In section 3, the structure of the light scalar mesons N was discussed. From the consideration of the mass spectrum of $m_{a_0(980)} \sim m_{f_0(980)} > m_{\kappa(900)} > m_{\sigma(600)}$ and the $s\bar{s}$-like character of $f_0(980)$, we concluded that the configuration of the light scalar $I = 0$ mesons are $f_0(980) \sim \frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us)$ and $\sigma(600) \sim \bar{u}\bar{d}ud$. We considered the intra-mixing using the effective Lagrangian expressed as $L^{eff} = -a\text{Tr}(NN) - b\text{Tr}(NNM) - \frac{1}{2}\lambda\text{Tr}(N)\text{Tr}(N)$, in which the last term corresponds to the 2nd order OZI-suppression term. The mixing among the $L = 1$ $q\bar{q}$ nonet is expressed by the same effective Lagrangian and the glueball mixing is also considered there. We consider that the $f_0(1500)$ is the most probable candidate of the glueball.

In section 4, we considered the overall inter- and intra-mixing among the light scalar nonet N, $L = 1$ $q\bar{q}$ nonet N' and glueball f_G. The effective Lagrangian of the inter-mixing

is assumed as $L_{01}^{eff} = \lambda_{01}(\mathrm{Tr}(NN') - \mathrm{Tr}(N)\mathrm{Tr}(N'))$. From the best fit analysis of χ^2, we obtained the results: $\lambda_{01} = 0.51$ GeV^2, $\lambda_0 = 0.05$ GeV^2, $\lambda_1 = 0.05$ GeV^2, $\lambda_G = 0.26$ GeV^2, $\lambda_{GG} = 1.53$ GeV^2, $m_{f_0(980)} = 0.981$ GeV, $m_{\sigma(600)} = 0.455$ GeV, $m_{f_0(1370)} = 1.376$ GeV, $m_{f_0(1710)} = 1.715$ GeV and $m_{f_0(1500)} = 1.499$ GeV. Obtained mixing parameters are also consistent with the experimental characters.

References

[1] J. Schwinger, Ann. of Phys. **2**,407(1957);
M. Gell-Mann and M. Levy, Nuovo Cim. **16**, 705(1960).

[2] N. A. Törnqvist and M. Roos, Phys. Rev. Lett. **76**, 1575(1996);
M. Harada *et al.*, Phys. Rev. **D54**, 1991(1996);
S. Ishida, M. Y. Ishida, H. Takahashi, T. Ishida, K. Takamatsu and T. Tsuru, Prog. Theor. Phys. **95**, 745(1996);
S. Ishida, T. Ishida, M. Y. Ishida, K. Takamatsu and T. Tsuru, *ibid.* **98** 1005(1997);
S. Ishida, M. Y. Ishida, T. Ishida, K. Takamatsu and T. Tsuru, *ibid.* **98** 621(1997).

[3] J. Schechter and Y. Ueda, Phys. Rev. **D3**, 2874(1971);
M. Ishida, Prog. Theor. Phys. **101**, 661(1999).

[4] N. Isgur and J. Weinstein, Phys. Rev. Lett. **48**, 659(1982).

[5] R. L. Jaffe, Phys. Rev. D**15**, 267(1977).

[6] N. N. Achasov, Nucl. Phys. **A675**, 279c(2000);
D. Black, A. H. Fariborz, F. Sannio, and J. Schechter, Soryushiron Kenkyu (Kyoto) **102**, E115(2001).

[7] H. Fritzsch and P. Minkowski, Nuovo Cim. **30A**, 393(1975);
P. G. O. Freund and Y. Nambu, Phys. Rev. Lett. **34**, 1645(1975).

[8] C. Amsler and F. Close, Phys. Lett. B**353**, 385(1995);

V. V. Anisovich, D.V. Bugg, and A. V. Sarantsev, Phys. Rev. D **58**, 111508(1998);

M. Jaminon and B. Van den Bosche, Nucl. Phys. A **619**, 285(1997);

W. Lee and D. Weingarten, Phys. Rev. D **59**, 094508(1999).

[9] Particle Data Group, Eur. Phys. Jour. **C15**, 1(2000) (URL : http://pdg.lbl.gov).

[10] D. Black, A. H. Fariborz, F. Sannio, and J. Schechter, Phys. Rev. D**59**, 074026(1999).

[11] T. Teshima and S. Oneda, Phys. Rev. D **33**, 1974(1986);

T. Teshima, I. Kitamura and N. Morisita, Il Nuovo Cim. **103**A, 175 (1990).